STAT 315: Uniform Random Variables

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Uniform Random Variables

A continuous random variable U is uniform if

- U takes values in some interval $[\theta_1, \theta_2]$.
- The probability that U takes value in some sub-interval $[a, b]$ (contained in $[\theta_1, \theta_2]$) only depends on the length of the interval $b - a$.

The PDF is constant on the interval $[\theta_1, \theta_2]$

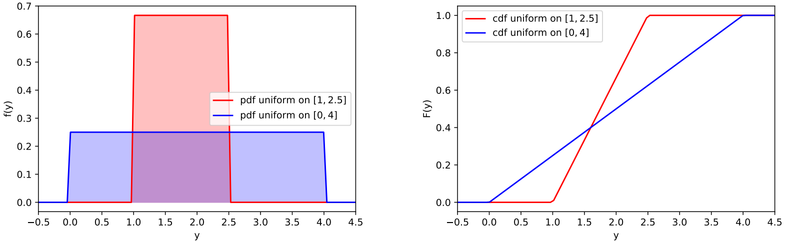

Figure: The PDF and CDF of 2 uniform RV on $[0, 4]$ and $[1, 2.5]$

PDF and CDF for uniform RV

PDF and CDF for the Uniform RV

If U is uniform on $[\theta_1, \theta_2]$ then the PDF is constant on $[\theta_1, \theta_2]$:

$$f(y) = \begin{cases} \frac{1}{\theta_2 - \theta_1} & \text{if } \theta_1 \le y \le \theta_2 \\ 0 & \text{otherwise} \end{cases}$$

$$P(a \le Y \le b) = \int_a^b \frac{1}{\theta_2 - \theta_1} dy = \frac{b-a}{\theta_2 - \theta_1}$$

The CDF is given by

$$F(y) = \begin{cases} 0 & \text{if } y \le \theta_1 \\ \int_{\theta_1}^{y} \frac{1}{\theta_2 - \theta_1} dy = \frac{y - \theta_1}{\theta_2 - \theta_1} & \text{if } \theta_1 \le y \le \theta_2 \\ 1 & \text{if } y \le \theta_2 \end{cases}$$

Notation: Write $X \sim U([\theta_1, \theta_2])$. Often use the letter U if $U \sim U([0,1])$

Mean and Variance of uniform RV

Mean and Variance

If U is uniform on the interval $[\theta_1, \theta_2]$

$$E[U] = \frac{\theta_1 + \theta_2}{2} \qquad V[U] = \frac{(\theta_2 - \theta_1)^2}{12}$$

Random number generator

Any computer system contains a (pseudo-random) number generator which is an algorithm (hard one!) which generates observed values for a uniform random variable $U \sim U([0,1])$. Usually the command is "rand"

```
# prompt: random number generator

import numpy as np
import random

np.random.rand(10)
```

```
array([0.32006209, 0.41149266, 0.6831682 , 0.2066693 , 0.22082369,
       0.75809193, 0.59138499, 0.55586962, 0.49877951, 0.76689334])
```

This is the only source of randomness on your computer and all random variable simulated on computer are derived from "rand". (More on this in Chapter 6)

Example: To flip a fair coin on a computer do the following:
if $U \leq \frac{1}{2}$ return 'Tails'
else return 'Heads'.